

# Multigrid methods applied to the Helmholtz equation

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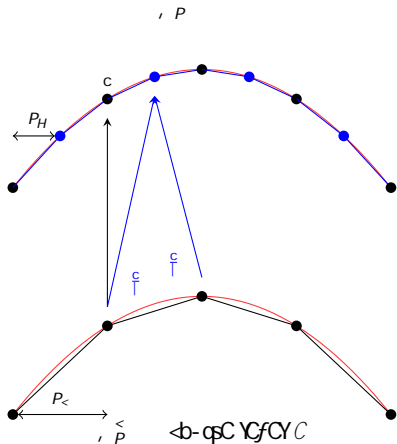
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# 1-Introduction

[ bsz zqfS YS^zCqpbY zbq =



Interpolation from coarse space to original space is made by

$$d_c = \frac{1}{2} \begin{pmatrix} 2 & \dots & 0 \\ 1 & \dots & 3 \\ 6 & & 7 \\ 2 & & 7 \\ 1 & 1 & 7 \\ 0 & 2 & \dots & 7 \\ 0 & 0 & \dots & 7 \\ 4 & 1 & \dots & 5 \\ 0 & \dots & \dots & \end{pmatrix}$$

# 1 - Introduction

$$(R^{\wedge}C^{\wedge} \wedge S^{\wedge}C^{\wedge}O^{\wedge}C^{\wedge}P^{\wedge}b^{\wedge}z^{\wedge} \ d^{\wedge}p^{\wedge}4^{\wedge}Y^{\wedge}C^{\wedge}) \quad , \quad \begin{matrix} \sim & W^{\sim} = H^{\wedge}b^{\wedge} \\ @^{\wedge}y^{\sim} = & S^{\wedge}b^{\wedge} @ \end{matrix}$$

y PC @S<qZS -zS^ bH YC @s zb -^ S^@C^ ^S^C s%szC^ , ~ = H^ S^fby^S^L z.b \ -U^bq Ss~Cs  
 Hbq \ ~Y^S.cq@ \ CzPb@s =

- B^S^C^f^ -Y^Cs -q^ 4bzP sS^ ^C@ ) d^p^4^Y^C^ -z^S^ Hbq^s^ \ bbzP^S^L^ szCes
- a s<S^Y^ zbq^%G^ q^V^C^q^C^Y^se^ <C ) O^ -q^@^ zb \ -V^C^ -eedq^e^q^S^ zC^ S^z^C^q^e^b^Y^ zbq^s

y -q -Cz =GS^@s^ \ bbzPCq^s - ^@ S^z^C^q^e^b^Y^ zbq^s \ -V^S^L^ \ ~Y^S.cq@ \ CzPb@s <b^f^C^q^L^S^L^ S^ -  
 <b^sz^ -^z^ ^~\ 4Cq^b^H^S^C^q^ z^S^s - ^@ S^@C^e^C^@C^z^%b^H^W

- 1 r \ bbzPCq = ] bq \ -Y^C^ --z^S^s^ \ CzPb@s bqV^q^%b^f^ S^C^q^ z^S^s^
- 2 R^z^C^q^e^b^Y^ zbq = **Still an open question ...**

# 1 - Introduction

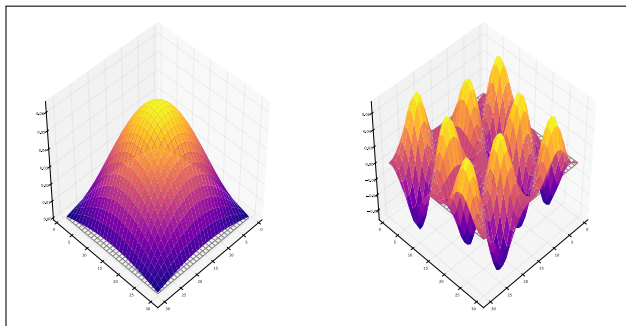


Figure: Laplace ( $W=0$  -  $s \setminus bbzP$ ) / Helmholtz ( $W \neq 0$  -  $\dots f \cos \omega$ ) near-kernels

Reminder : Near-kernel space is defined by the set of eigenvectors associated to smallest absolute eigenvalues

! These eigenvectors are the most important ! ,  $c_4 = \prod_{s \setminus \lambda_s} \alpha_s f_s$

k @ TT`QtBK iBM; i?2 A/2 H AMi2`

G2i M MK i`Bt r?2`2(`)M R<sup>M</sup>-t M/#`2bT2+iBp2Hv bQHmiBQ  
bB/2 Q7 i?2 ttv=b#2K

A/2 H B Mi2 ST Q b K Q biHv mb2/ 7Q` i?2Q`2iB+ H Tm`TQb2-  
iBQM QM i?2 +QM p2`;2M+2 b+2 M(+BQ)=FQ FBNQ2BTM;Bi;BM2X

6B;m82M/ N TQB MiC=Fa i2MHBHBM;b

k @ TT`QtBK iBM; i?2 A/2 H AMi2`

G2a M/\_h #Qi? BMD2+iBQM BMi2`TQH iQ`b bm+? i? i

$$C[F \ 7^h \ ] ; C[F \ 7C$$

AM i?2 HBi2` im`2- i?2 A/2 H AMi2`TQH iQ` Bb /2}M2/ i?2

$$S = (A \ a(a^h \ a) \ R^h \ a) \ _h$$

G2i`2Q`; M/Bx+2? i? i

$$= \begin{matrix} 77 & 7+ \\ +7 & ++ \end{matrix} ; M/a = \frac{A7}{y} ; \ _h = \frac{y}{A+}$$

$$h?m\$ = \begin{matrix} R \\ 77 & 7+ \\ A+ & \end{matrix} M/ += S^h \ S = \frac{\begin{matrix} ++ \\ \{77\} \\ a+?m` *QKTH2K2Mib 7Q`K mH \end{matrix}}{\begin{matrix} 7+ \\ 6 \\ - \end{matrix}} \ _h = \begin{matrix} ++ \\ \end{matrix}$$

S `2KQp2b i?2 r?QH2 }M2 `2H i2/ BM7Q`K iB  
`2T`2b2Mi iBQM 5



k @ TT`QtBK iBM; i?2 A/2 H AMi2`

S`Q#H\$K+,QMi BMb M 2t +i BMP2`bBQM- r?B+? BBbiQ2M2  
BirBH H HBKBi Qm` + T +Biv iQ +Q `b2M /2  
"mi Bi Bb biBHH TQbbB #h2 a)Q<sup>R</sup> #T(a<sup>h</sup>Qa)B<sup>R</sup>5i2

6B;m`T2T`QtBK iBQMa)Q<sup>R</sup>mbBM; a+?m` +QKTH2K2M  
h?2M r2 /2}M2 TT`QtBK iBQM Q7 A/2 H AMi2`TQH iQ` b

$$S = \left( \frac{A \alpha \{a^h, a\} R a^h}{F_{-2H} t i BQM} \right)^{-h}$$

k @ TT`QtBK iBM; i?2 A/2 H AMi2`

$$G \Xi = (a^h a)^R (a^h a)^R = \begin{matrix} R \\ 77 \end{matrix} \begin{matrix} R \\ 77 \end{matrix} X h?mb Bi 7QHHQrb i? i$$

$$S = \begin{matrix} R \\ 77 \end{matrix} \begin{matrix} R \\ 77 \end{matrix} \begin{matrix} R \\ 77 \end{matrix} E \begin{matrix} R \\ 77 \end{matrix} = \begin{matrix} S \\ A/2 H AMi2`T H(z) \\ LQBb2 \end{matrix} + \begin{matrix} E \\ y \\ P(z) \end{matrix} \begin{matrix} R \\ 77 \end{matrix}$$

:Q H6,BM/ ;QQ/ i` /2@Qz #2ir22M bT `bBiv M/ MQ

>Qr iQ `2KQp2 i?2 MQBb2 r?B+? /2;` /2b i?2 +Q `b2 `2T  
bT +2\

A/2 //BM; +Q``2+iBQM K i`Bt iQ i?2 B/2 H TT`QtBK iBQ

$$S = (A s^R)(A \alpha(a^h a)^R a^h)_h$$

\_2K`Fq2 M22/ iQ F22T BMSKB?M/m H/i#2 i?2 bT `b2bi TQbbE  
J b?QmH/ MQi / K ;2 i?2 M2 `@F2`M2H bT +2s?BH2`2KQ

j @ h2Mi iBp2 BMi2`TQH iQ` #mBH

h?2 7QHHQrBM; B/2 Bb BMbTB`2/ #v i?2 aKQQi?2/ ;;`2;

>2`2 Bb Bib T`BM+BTH2 ,

R G2i bvbib2K #rBi? FMQrM bQHmiBQM=UyQX BMbi M+2

k TT`QtBK #2# rBi? 72r bKQQi?BM; Bi2` iBQ2#bt- i?2M +0

j \*QMbi`m2iMi iBp2 BMi2`TQH? iQ`iT 2,T h 2= 2X

9 h?2M +QKS=miJZ rBi? bQK2 2``Q`T`QT ; iBQM K i`BtX

) Si `;2ib `2K BMBM; BM? QbK QQQM2` Bb MQi #H2 iQ + T



j @ h2Mi iBp2 B Mi2`TQH iQ` #mBH i

> Qr iQ + Q Mtbv`m+i

- RX 6`QK CFP 2MHBiiBM;- B Mp B;/;2HQK2` i2b M/ +QKTm  
HQR2bi +QKT QM Mi

> Qr iQ + Q MTbvi`m+i

- $k X 6 Q` 2 + ? ; ; H Q K 2 Q K T m i 2 i ? 2 > Q m b 2 ? Q H / 2` ` 2 ~ 2 + i B Q M$   
 $Z_B^h p(A_B) = k p(A_B) k k n_B^{(C)} r B i n_B^{(C)} + M Q M B + H p 2 (Q) X` a B M + B b$   
 $Z_B^h p(A_B) B b M m H H (Q) M 2 H 2 K 2 M i b - F 2 2 T Q Q Y K Z B X Q H m K M$

$$Z_B^h = \left( A \quad k \frac{p(A_B) p(A_B)^h}{k p(A_B) k k} \right)$$

M /

$$Z_B^h p(A_B) = k p(A_B) k k n_B^{(C)}$$

$$p(A_B) = Z_B^h p(A_B) k k n_B^{(C)}$$

6 B ; m > Q m b 2 ? Q H / 2` ` 2 ~ 2 + i B Q M

j @ h2Mi iBp2 B Mi2`TQH iQ` #mBHi

- jX \_2T2 i i?2 T`Q+2A b 7IQ/ #m+B?H/ i?2 #HQ+FT+QHmKM

j @ h2Mi iBp2 B Mi2`TQH iQ` #mBHi

hQ bmKK `Bx2 i?2 +Q Mbi`m+iBQM Q7

$$S = (A \quad r. \quad R) (A \quad a \quad a^h \quad a) \quad R a^h \quad )T$$



9 @ " 2 M + ? K ` F b

6 B ; m A 22 H p b X a K Q Q i ? 2 / A / 2 H " H Q + F T T ` Q t B K i l

9 @ " 2 M + ? K ` F b

6 B ; m Å 22 H p b X a K Q Q i ? 2 / A / 2 H " H Q + F T T ` Q t B K i l

## 5 - To do next

- 1 Use Conjugate Gradient on Normal Equations (CGNR) instead of ...-Jacobi as smoothing matrix
- 2 Add constraint in CGNR sub-research space to keep interesting properties in coarse matrices in order to coarsen deeper. (structure, clean near-kernel space, etc.)