

Comparing mixed-precision solving with low-rank compression

Working Group

Brieuc NICOLAS

January 13th 2023

Solving linear systems

Direct resolution with LU factorization

$$A = LU, L = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ l_1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ l_2 & \cdots & l_3 & 1 \end{pmatrix} \text{ et } U = \begin{pmatrix} u_1 & \cdots & \cdots & u_2 \\ 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & u_3 \end{pmatrix}$$

$$Ax = b$$

$$LUx = b$$

$$Ly = b$$

$$y = Ux.$$

Context of the assignment

Comparison of mixed-precision solving in relation to standard precision and low-rank :

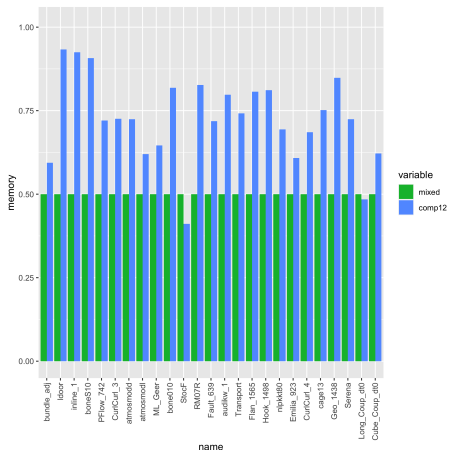
- ① Time: Mixed-precision should be faster as there are theoretically half the operations to make
- ② Numerical precision: The trade is that converging towards a solution should be harder
- ③ Memory usage: There should be half the memory usage compared to normal
- ④ Energy consumption: The energy consumption should be lower but the number of iterations must be taken into consideration

Parameters

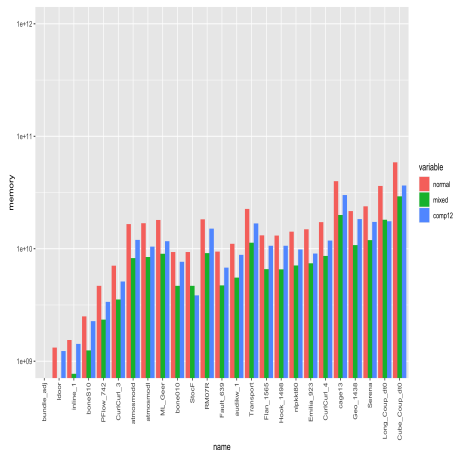
PaStiX parameters that were used for the following measurements :

	normal	mixed	low-rank
Magnitude controle	$1e^{-15}$	$1e^{-7}$	$1e^{-15}$
Compression tolerance			$1e^{-12}$
refinement epsilon	$1e^{-12}$		
block size	256-512		

Memory Usage



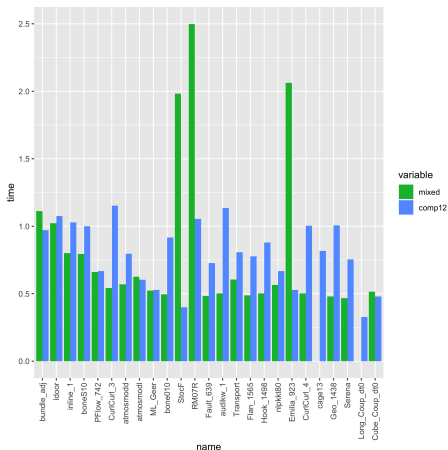
(a) Memory usage ratio



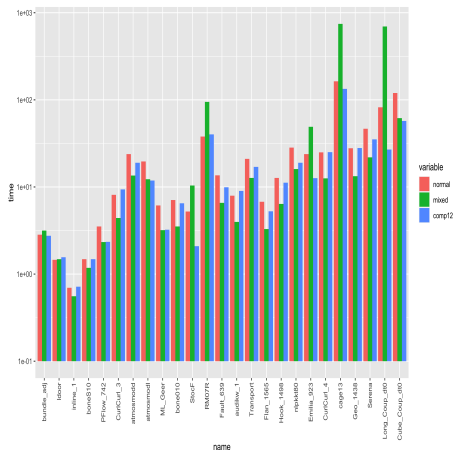
(b) Memory usage, log scale

Figure: Comparison of the memory usage in normal, mixed precision and low-rank

Factorization



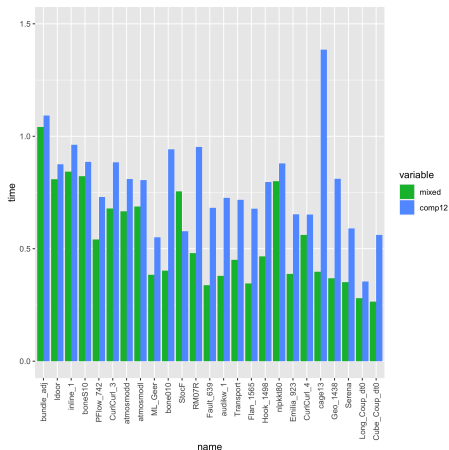
(a) Factorization time ratio



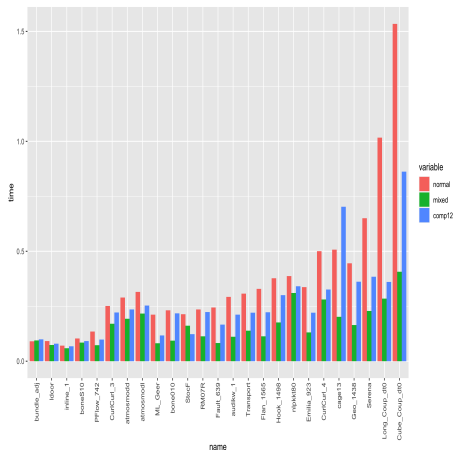
(b) Factorization time, log scale

Figure: Comparison of the time to factorize in normal and mixed precision and low-rank

Solving step



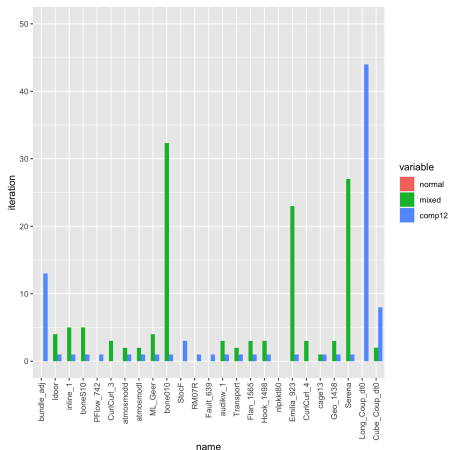
(a) Solving time ratio



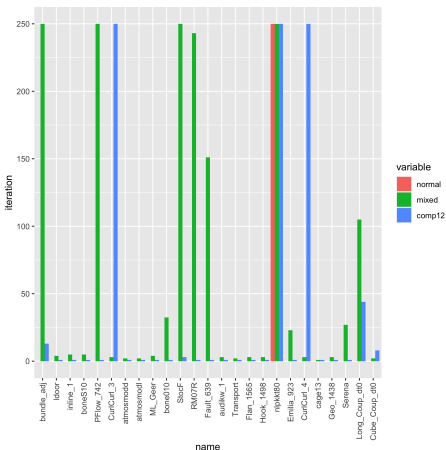
(b) Solving time

Figure: Comparison of the time to solve in normal, mixed precision and low rank

Refinement



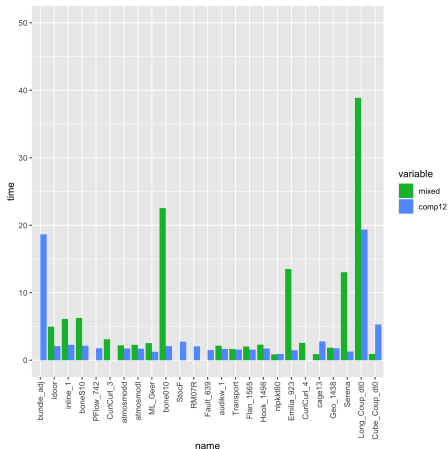
(a) Iteration number close-up



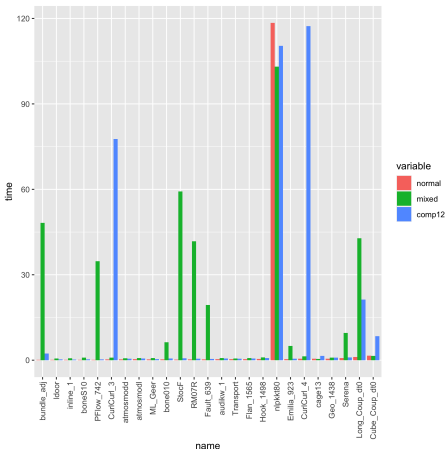
(b) Iteration number

Figure: Comparison of the number of iterations in normal, mixed precision and low-rank

Refinement



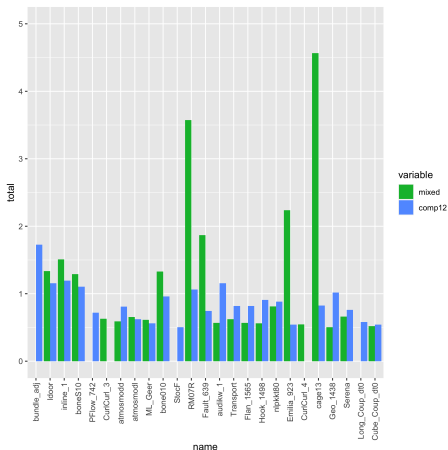
(a) Refinement time ratio



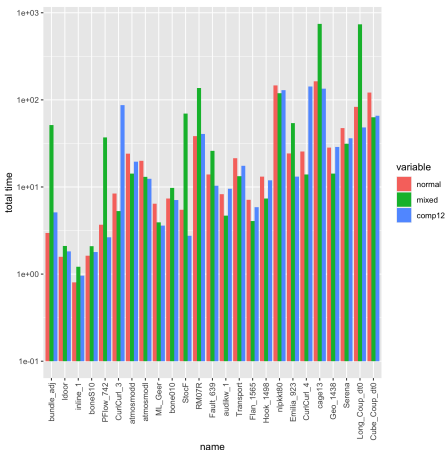
(b) Refinement

Figure: Comparison of the time to refine in normal, mixed precision and low-rank

Total



(a) Total time ratio



(b) Total time, log scale

Figure: Comparison of the total time in normal, mixed precision and low-rank

Conclusion

Work to be done:

- 1 Refining those results (there are some stand-outs)
- 2 Evaluating the energy consumption