Comparing mixed-precision solving with low-rank compression

Working Group

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Solving linear systems

Direct resolution with LU factorization

\[ A = LU, \quad L = \begin{pmatrix}
1 & 0 & \cdots & 0 \\
\cdot & \ddots & \ddots & \vdots \\
\cdot & \cdot & \ddots & 0 \\
\cdot & \cdot & \cdot & 1
\end{pmatrix}, \quad \text{et} \quad U = \begin{pmatrix}
u_1 & \cdots & \cdots & u_2 \\
0 & \ddots & \vdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & u_3
\end{pmatrix}\]

\[ Ax = b \]
\[ LUx = b \]
\[ Ly = b \]
\[ y = Ux. \]
Comparison of mixed-precision solving in relation to standard precision and low-rank:

1. **Time**: Mixed-precision should be faster as there are theoretically half the operations to make.
2. **Numerical precision**: The trade is that converging towards a solution should be harder.
3. **Memory usage**: There should be half the memory usage compared to normal.
4. **Energy consumption**: The energy consumption should be lower but the number of iterations must be taken into consideration.
PaStiX parameters that were used for the following measurements:

<table>
<thead>
<tr>
<th></th>
<th>normal</th>
<th>mixed</th>
<th>low-rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude control</td>
<td>$1e^{-15}$</td>
<td>$1e^{-7}$</td>
<td>$1e^{-15}$</td>
</tr>
<tr>
<td>Compression tolerance</td>
<td></td>
<td></td>
<td>$1e^{-12}$</td>
</tr>
<tr>
<td>refinement epsilon</td>
<td></td>
<td></td>
<td>$1e^{-12}$</td>
</tr>
<tr>
<td>block size</td>
<td></td>
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<td>256-512</td>
</tr>
</tbody>
</table>
Memory Usage

(a) Memory usage ratio

(b) Memory usage, log scale

Figure: Comparison of the memory usage in normal, mixed precision and low-rank

Brieuc NICOLAS
Comparing mixed-precision solving with low-r; January 13th 2023 5 / 11
Figure: Comparison of the time to factorize in normal and mixed precision and low-rank
Figure: Comparison of the time to solve in normal, mixed precision and low rank

(a) Solving time ratio

(b) Solving time
Refinement

(a) Iteration number close-up

(b) Iteration number

Figure: Comparison of the number of iterations in normal, mixed precision and low-rank
Refinement

(a) Refinement time ratio

(b) Refinement

Figure: Comparison of the time to refine in normal, mixed precision and low-rank
Figure: Comparison of the total time in normal, mixed precision and low-rank
Conclusion

Work to be done:

1. Refining those results (there are some stand-outs)
2. Evaluating the energy consumption