

*Inria*

# PaStiX: Distributed Interface

Supervisors: Mathieu Faverge  
and Pierre Ramet

Alycia Lisito

# Summary

01. PaStiX
02. Matrix permutation
03. Vector permutation
04. Performances
05. Conclusion

# 01

PaStiX

### PaStiX = Parallel Sparse Linear Algebra Solver

- Linear Algebra Solver
  - > Solves  $Ax = b$
- Sparse
  - > Matrix with a lot of zero elements
- Parallel
  - > Several schedulers:
    - Sequential
    - Static
    - Dynamic
    - StarPU
    - Parsec
  - > MPI

### 4 steps

- Analyse
  - > Permutation  $P$
  - > Blocks
- Numerical Factorisation
  - >  $\mathbf{A} \rightarrow \mathbf{PAP}^T \rightarrow LU$
- Solve
  - >  $\mathbf{b} \rightarrow \mathbf{Pb}$
  - > Solves  $Ly = Pb$
  - > Solves  $UPx = y$
  - >  $\mathbf{Px} \rightarrow \mathbf{x}$
- Refinement
  - > Refines the solution  $x$

## The matrix format: CSC format

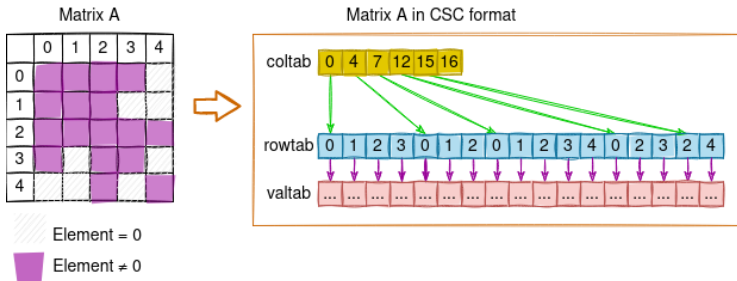


Figure: Example of a matrix  $A$  in the CSC format ( $A_{CSC}$ )

## The CSC format in distributed memory

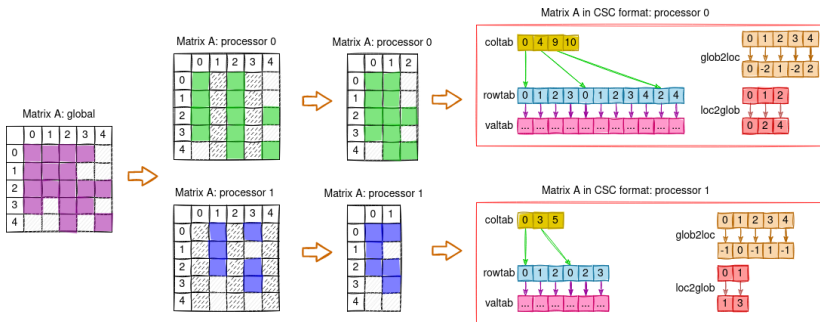


Figure: Example of a **distributed** matrix  $A$  in the CSC format ( $A_{CSC}$ )

## The block format: *BCSC* format

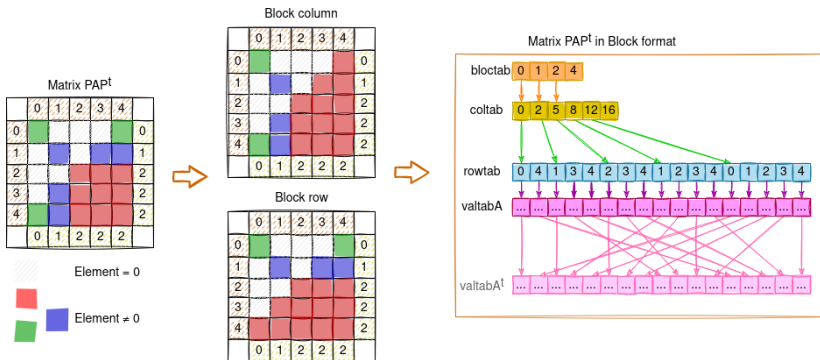


Figure: Example of a matrix  $PAP^T$  in the *BCSC* format ( $PAP^T_{BCSC}$ )



## Degree of Freedom: Single

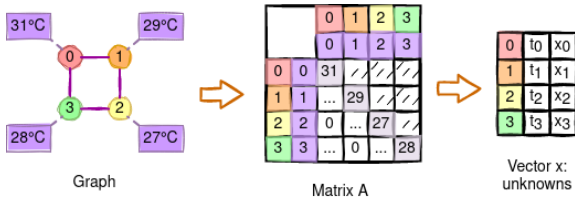


Figure: Example of a matrix  $A$  with a Single DoF

## Degree of Freedom: Multiple constant

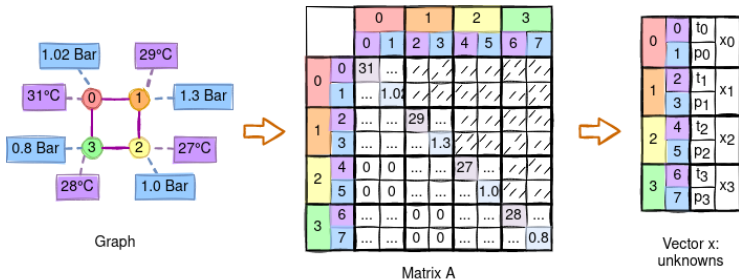


Figure: Example of a matrix  $A$  with a Multiple Constant DoF

## Degree of Freedom: Multiple variadic

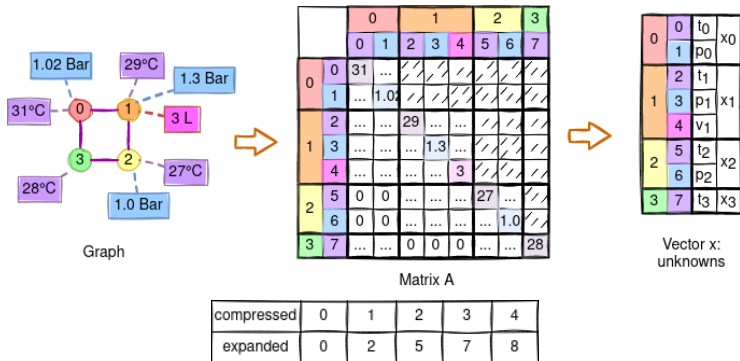


Figure: Example of a matrix A with a Multiple Variadic DoF

# Goal of my work

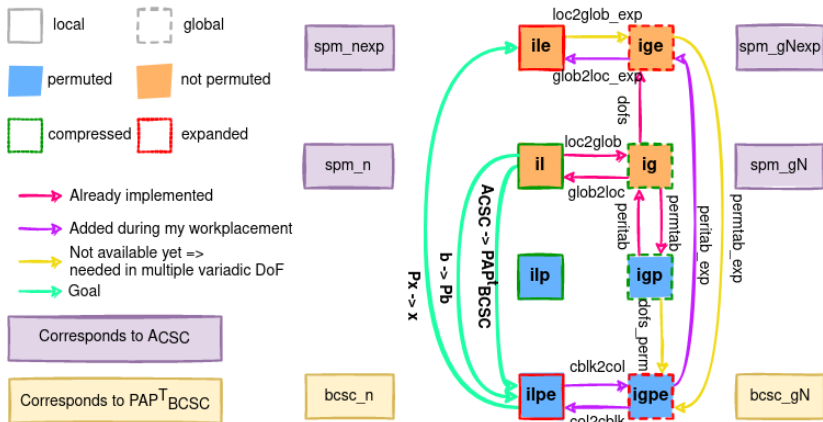


Figure: The different types of indexes

# 02

## Matrix permutation

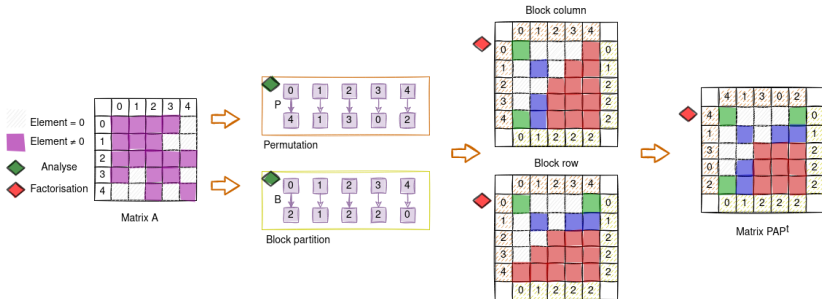
Matrix permutation:  $A \rightarrow PAP^T$ 

Figure: Goal of the permutation and block storage

Matrix A: processor 0

|   |   |   |   |
|---|---|---|---|
|   | 0 | 2 | 4 |
| 0 | █ | █ | █ |
| 1 | █ | █ | █ |
| 2 | █ | █ | █ |
| 3 | █ | █ | █ |
| 4 | █ | █ | █ |

Matrix A: processor 1

|   |   |   |
|---|---|---|
|   | 1 | 3 |
| 0 | █ | █ |
| 1 | █ | █ |
| 2 | █ | █ |
| 3 | █ | █ |
| 4 | █ | █ |

Initial data



Matrix  $PAP^T$ : block columns

|   |   |   |   |   |   |
|---|---|---|---|---|---|
|   | 4 | 1 | 3 | 0 | 2 |
| 4 | █ | █ | █ | █ | █ |
| 1 | █ | █ | █ | █ | █ |
| 3 | █ | █ | █ | █ | █ |
| 0 | █ | █ | █ | █ | █ |
| 2 | █ | █ | █ | █ | █ |
|   | 0 | 1 | 2 | 2 | 2 |

Final data

Matrix  $PAP^T$ : block columns local

|   |   |   |   |   |   |
|---|---|---|---|---|---|
|   | 4 | 1 | 3 | 0 | 2 |
| 4 | █ | █ | █ | █ | █ |
| 1 | █ | █ | █ | █ | █ |
| 3 | █ | █ | █ | █ | █ |
| 0 | █ | █ | █ | █ | █ |
| 2 | █ | █ | █ | █ | █ |
|   | 0 | 1 | 2 | 2 | 2 |

Final data: local data

Matrix  $PAP^T$ : block columns remote

|   |   |   |   |   |   |
|---|---|---|---|---|---|
|   | 4 | 1 | 3 | 0 | 2 |
| 4 | █ | █ | █ | █ | █ |
| 1 | █ | █ | █ | █ | █ |
| 3 | █ | █ | █ | █ | █ |
| 0 | █ | █ | █ | █ | █ |
| 2 | █ | █ | █ | █ | █ |
|   | 0 | 1 | 2 | 2 | 2 |

Final data: remote data

Matrix  $PAP^T$ : block rows

|   |   |   |   |   |   |
|---|---|---|---|---|---|
|   | 4 | 1 | 3 | 0 | 2 |
| 4 | █ | █ | █ | █ | 0 |
| 1 | █ | █ | █ | █ | 1 |
| 3 | █ | █ | █ | █ | 2 |
| 0 | █ | █ | █ | █ | 2 |
| 2 | █ | █ | █ | █ | 2 |

Matrix  $PAP^T$ : block rows local

|   |   |   |   |   |   |
|---|---|---|---|---|---|
|   | 4 | 1 | 3 | 0 | 2 |
| 4 | █ | █ | █ | █ | 0 |
| 1 | █ | █ | █ | █ | 1 |
| 3 | █ | █ | █ | █ | 2 |
| 0 | █ | █ | █ | █ | 2 |
| 2 | █ | █ | █ | █ | 2 |

Matrix  $PAP^T$ : block rows remote

|   |   |   |   |   |   |
|---|---|---|---|---|---|
|   | 4 | 1 | 3 | 0 | 2 |
| 4 | █ | █ | █ | █ | 0 |
| 1 | █ | █ | █ | █ | 1 |
| 3 | █ | █ | █ | █ | 2 |
| 0 | █ | █ | █ | █ | 2 |
| 2 | █ | █ | █ | █ | 2 |

- data sent by 0 to 1
- data sent by 1 to 0
- data local to 0
- data local to 1

**Figure:** Data exchanged between the processor in the distributed memory case

## Processors communications: the difficulties

- How much data will I send ?
- How much data will I receive ?
- Where can I store the buffers ?

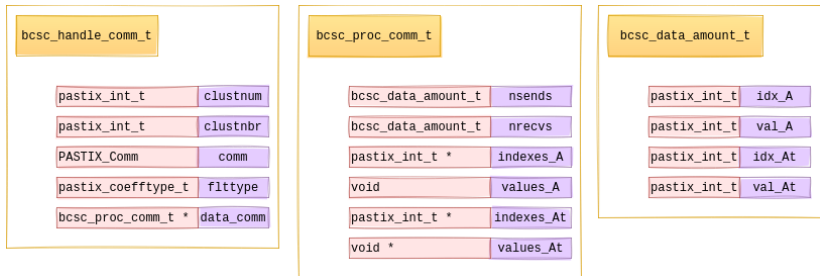


Figure: Structure to handle the processors communications



## How are the different indexes handled ?

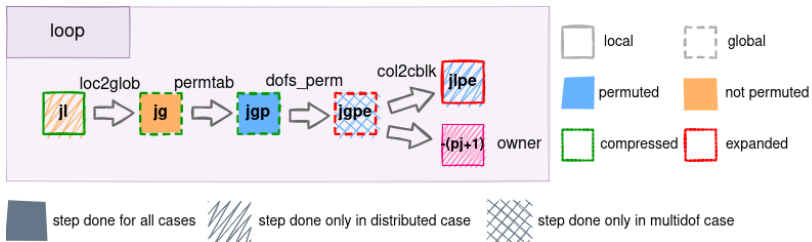


Figure: Algorithm in terms of indexes conversion

# 03

## Vector permutation

## Permutation of the vector: the replicated case

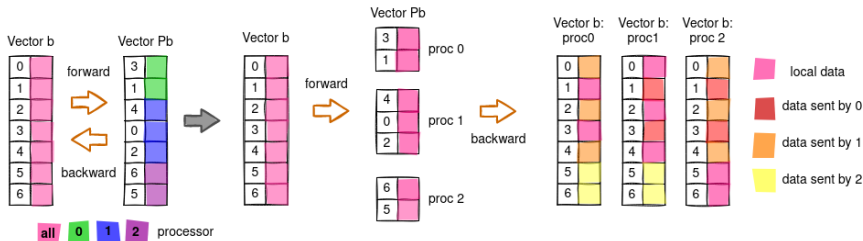


Figure: Data exchanged in the replicated case

## Permutation of the vector: the distributed case

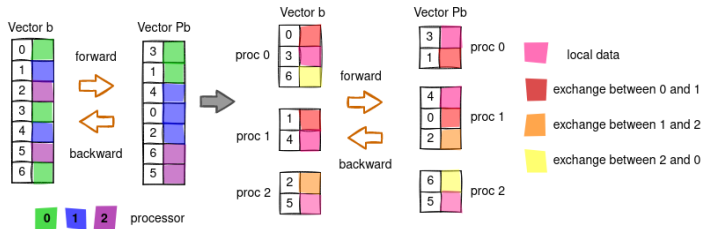
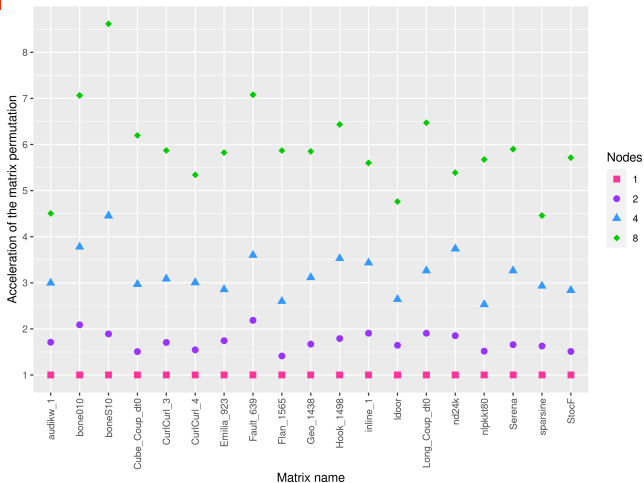


Figure: Data exchanged in the distributed case

# 04

## Performances

# Performances of the matrix permutation



**Figure:** Acceleration of the matrix permutation on 2, 4, and 8 nodes for different matrices

# 05

## Conclusion

### Goals achieved

- Matrix permutation in distributed memory
- Matrix permutation with multiple constant DoF
- Vector permutation in replicated to distributed case
- Vector permutation in distributed to distributed case
- Distributed sequential solve

### Next steps

- Improve the *MPI* communications
- Implement the distributed multi-threaded solve
- Implement the matrix permutation with variadic DoF
- Implement the vector permutation with variadic DoF