Sequential Scheduling of Dataflow Graphs for Memory Peak Minimization

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Context: DF4DL post-doc

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May compute guarantees on:

- liveness
- throughput
- memory usage
- real-time properties

Statically (e.g. SDF) or dynamically (e.g. RDF)

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Our memory peak problem

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A directed acyclic task graph, with memory costs:

- on each edge (data I/O)
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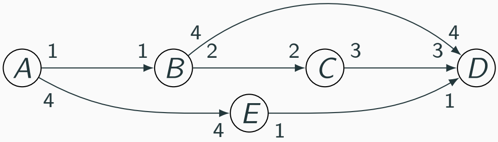
Output

- a schedule minimizing the memory peak
- its corresponding memory peak

Example



When to execute E ?

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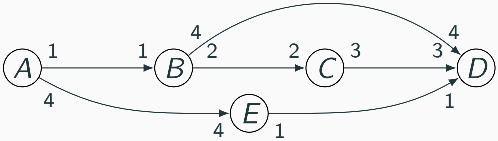

When to execute E **?** $A; E; B; C; D$

Previous known results

Complexity

[Sethi'73] PebbleGame is NP-complete (time)

[KS'74] Generate all Linear Extensions (linear in space)

[BW'91] Counting Linear Extensions is #P-complete ($\supseteq$ NP time)

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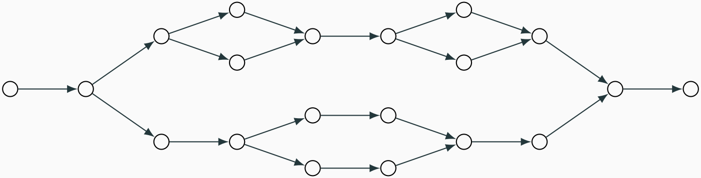
Specific cases

[Liu'87] trees in quadratic time

[KLMU'18] Series-Parallel DAG in cubic time

Example of an SP-DAG



(a sort of recursive fork-join graph)

“Optimal” graph transformations (contrib. 1)

Why graph transformations?

Key idea: reduce the combinatorial explosion by...

- reducing the number of nodes
- increasing the number of dependencies
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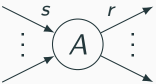
"Optimal" transformations
$\hookrightarrow$ preserve the minimal memory peak

From task graph to schedule graphs

Internal representation: node Peak and Impact



Node $A^{\binom{\text{peak}}{\text{impact}}}$ produces r tokens and consumes s tokens.

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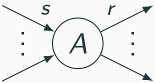

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Initial values of Peak and Impact
$\text{impact} = r - s \in \mathbb{Z}$ and $\text{peak} \in \mathbb{N}$, or peak variants:

- $A^{\binom{\max(0,r-s)}{r-s}}$ in the Consume-Before-Produce model
- $A^{\binom{r}{r-s}}$ in the Produce-Before-Consume model

(no further need to edge attribute)

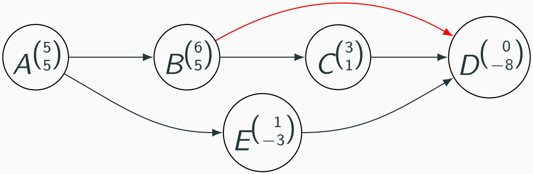
Peak of a schedule

Peak and Impact of a node sequence
Can be applied to any schedule.

$$A^{\binom{p_a}{i_a}}; B^{\binom{p_b}{i_b}} = (A; B)^{\binom{max(p_a, p_b + i_a)}{i_a + i_b}} \tag{PI}$$

Theorem
Operation (PI) is associative.

Transitive reduction



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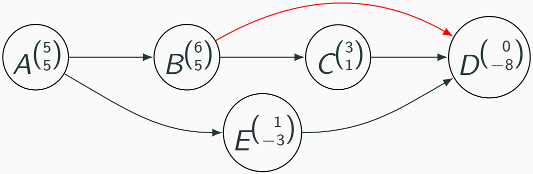

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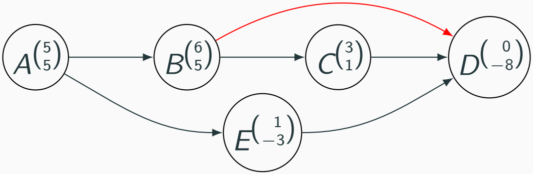

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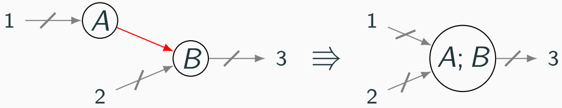
Does not modify node peak/impact!

Clustering rules (C1-C2): single successor/predecessor



$$\mathrm{Succ}(A) = \{B\} \;\wedge\; (i_A \geq 0) \;\wedge\; (p_B + i_A \geq p_A) \qquad \text{(C1)}$$

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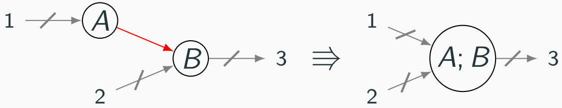

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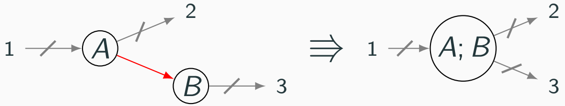
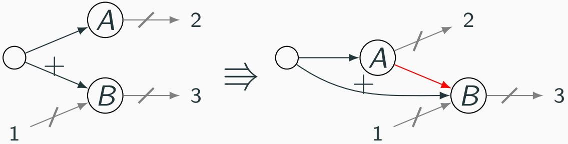


$$\mathrm{Pred}(B) = \{A\} \;\wedge\; (i_B \leq 0) \;\wedge\; (p_A \geq p_B + i_A) \qquad \text{(C2)}$$

Sequentialization rule (S1): common predecessors



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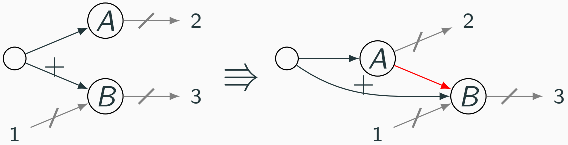

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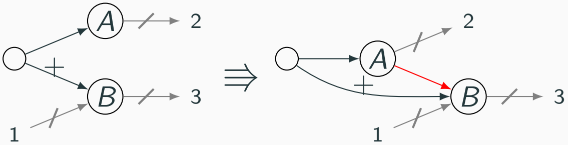

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Increase the number of dependencies!

(similar rule for common successors with Succ^{+})

Global algorithm

```
/* Takes a schedule graph G and compresses it until none of
   the transformations apply                                */
changed := false;
repeat
    repeat
        repeat
            clustering(G); ▷ O(n)
        until ¬ changed;
        basic_sequentialization(G); ▷ O(n^2)
    until ¬ changed;
    complete_sequentialization(G); ▷ O(n^3)
    transitive_reduction(G); ▷ O(n^3)
until ¬ changed;
```

Theoretical results

In general
Compressed graph always ensures at least one schedule having the minimal peak. (worst-case complexity: quartic time $\mathcal{O}(n^4)$)

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Compressed graph always ensures at least one schedule having the minimal peak. (worst-case complexity: quartic time $\mathcal{O}(n^4)$)

Specific cases
If reduced to a single node, it contains one of the schedule ensuring minimal peak. This includes:

trees compressed to a single node (in quadratic time)

SP-DAG compressed to a single node (in cubic time)

Branch and Bound search (contrib. 2)

General idea of Branch and Bound

Explore linear extensions of the graph...
New *branch* at each scheduled node

(storing all the ready unexplored ones),

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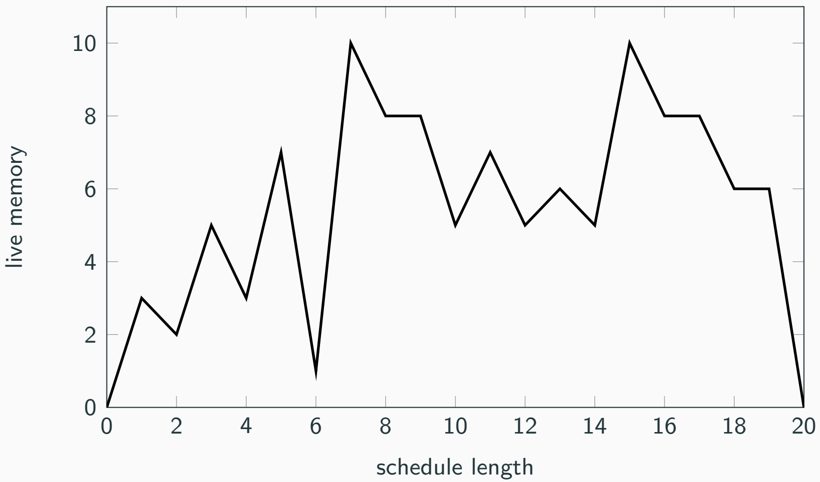
Optimizations (contributions)

- a longer backtrack
- a smaller ready list

Peak backtrack optimization



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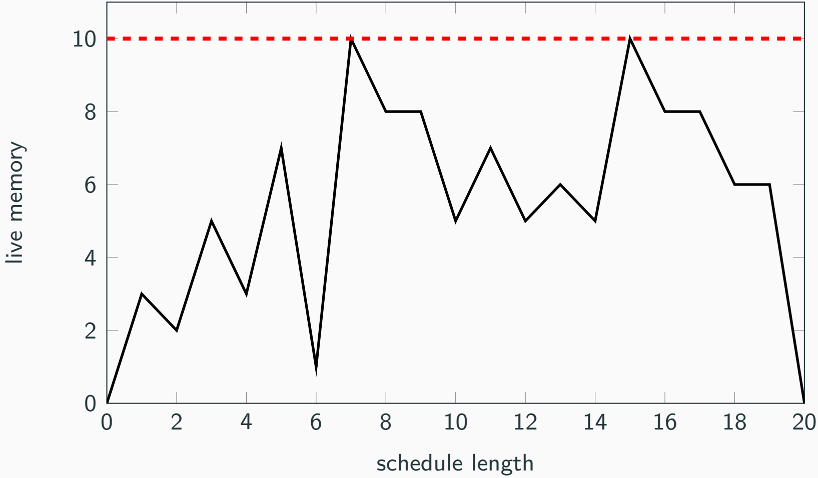

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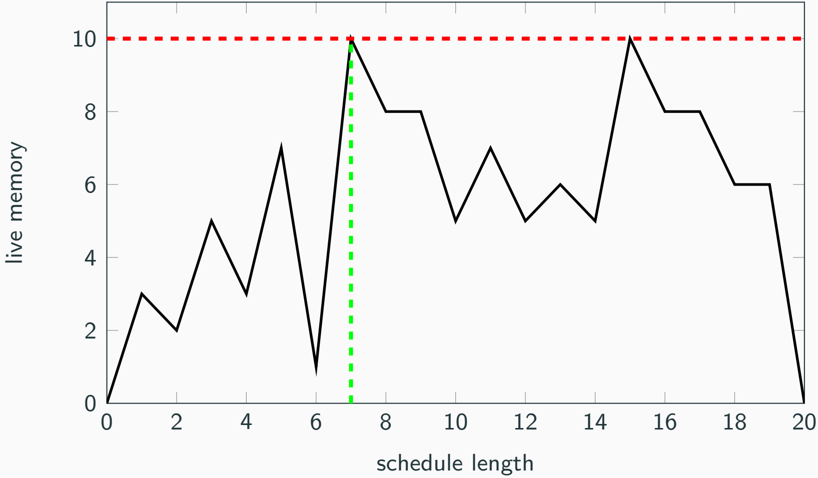
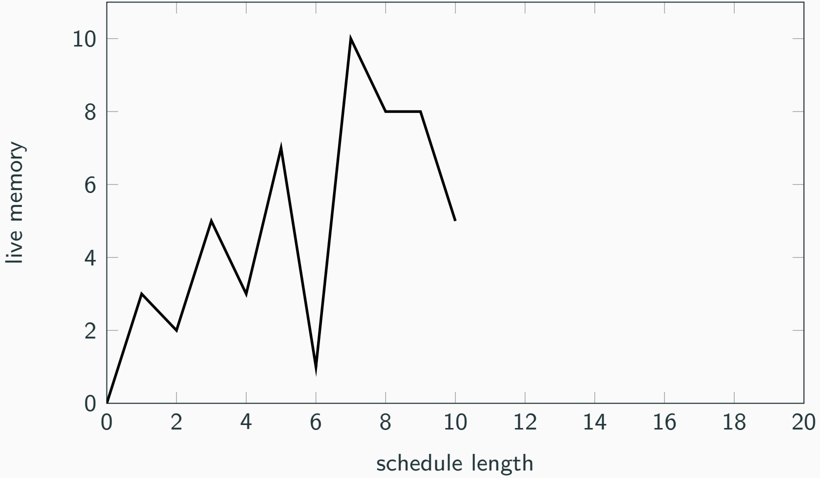


Backtrack until first peak!

Negative impact optimization



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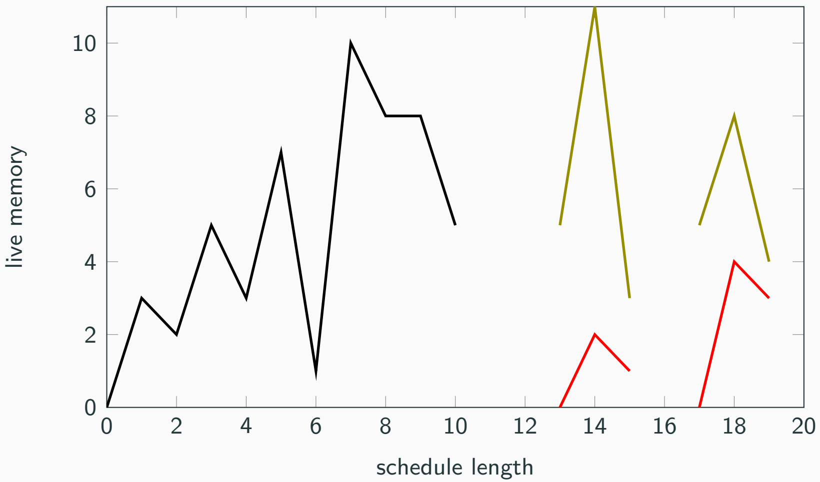

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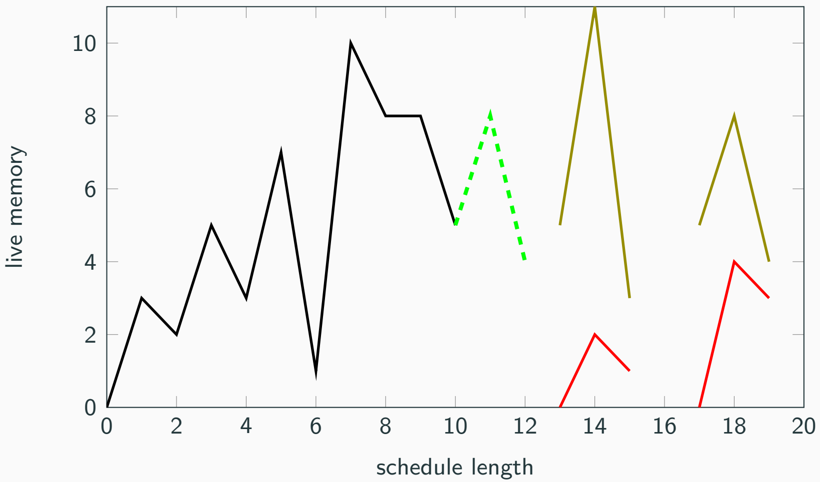

Negative impact node first!

(smaller branching factor)

Practical results

Instance size: our Branch and Bound
Graph of ≈ 50 nodes always solved in ≤ 1 sec.

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Important parameter: sort function for ready list
Will impact on the peak quality of first DFS.

Experiments

Memory peak for Satellite

satellite	$\|G\|$	[RWM'95]	[MB'01]	[KLMU'18]	[ours]	sec.
flat SAS	22	1,920	—	**1,680**	**1,680**	0.01
SDF	4,515	—	991	**960**	**960**	24.5

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Peaks of [MB'01] and [KLMU'18] are over-estimated.

Memory peak for QMF Filterbank

Filterbank	$\lvert G \rvert$	[MB'01]	[KLMU'18]	[ours]	$\lvert G^{\mathbf{C}} \rvert$	sec.
qmf23_2d	90	22	27	**14**	1	0.07
qmf23_3d	378	63	81	**32**	1	0.6
qmf23_5d	5,346	492	709	**248**	1	445.4
qmf12_2d	40	9	10	**7**	1	0.02
qmf12_3d	112	16	20	**11**	1	0.06
qmf12_5d	704	58	79	**35**	1	1.7
qmf235_2d	250	55	78	**24**	24	0.3
qmf235_3d	1,750	240	189	**47**†	285	T/O
qmf235_5d	68,750	5,690	—	**3,401**†	—	T/O

Gray = Wrong qmf version but similar peaks

(always reduced to 1 node on correct version, no T/O)

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Future work

- checkpointing a.k.a. rematerialization
  (a.k.a. reversible PebbleGame?)
- apply same kind of transformations to other problems?

References

[Sethi'73] *Complete Register Allocation Problems*, R. Sethi (1973)

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